

ASSOCIATED TENSORS.

37. If $A_j = \epsilon_{jk} A^k$, show that $A^k = \epsilon^{kj} A_j$.

Multiply $A_j = \epsilon_{jk} A^k$ by ϵ^{ij} .

Then $\epsilon^{ij} A_j = \epsilon^{ij} \epsilon_{jk} A^k = \delta^i_k A^k = A^i$, i.e. $A^i = \epsilon^{ij} A_j$ or $A^k = \epsilon^{kj} A_j$.

The tensors of rank one, A_j and A^k , are called *associated*. They represent the covariant and contravariant components of a vector.

38. (a) Show that $L^2 = \epsilon_{jk} A^j A^k$ is an invariant. (b) Show that $L^2 = \epsilon^{ij} A_i A_j$.

(a) Let A_j and A^k be the covariant and contravariant components of a vector. Then

$$\bar{A}_j = \frac{\partial x^i}{\partial \bar{x}^j} A_i, \quad \bar{A}^k = \frac{\partial \bar{x}^k}{\partial x^i} A^i$$

$$\text{and} \quad \bar{A}_j \bar{A}^k = \frac{\partial x^i}{\partial \bar{x}^j} \frac{\partial \bar{x}^k}{\partial x^i} A_i A^i = \delta^k_i A_i A^i = A_j A^j$$

so that $A_j A^j$ is an invariant which we call L^2 . Then we can write

$$L^2 = A_j A^j = \epsilon_{jk} A^k A^j = \epsilon^{ij} A_i A_j$$

(b) From (a), $L^2 = A_j A^j = A_j \epsilon^{jk} A^k = \epsilon^{jk} A_j A_k = \epsilon^{ij} A_i A_j$.

The scalar or invariant quantity $L = \sqrt{A_j A^j}$ is called the magnitude or length of the vector with covariant components A_j and contravariant components A^j .

39. (a) If A^p and B^q are vectors, show that $\epsilon_{pq} A^p B^q$ is an invariant.

(b) Show that $\frac{\epsilon_{pq} A^p B^q}{\sqrt{(A^i A_i)(B^j B_j)}}$ is an invariant.

(c) In Problem 38, $A^j A_j = \epsilon^{jk} A^k A^j = \epsilon^{ij} A_i A_j$ is an invariant.

(d) Since $A^p A_p$ and $B^q B_q$ are invariants, $\sqrt{(A^i A_i)(B^j B_j)}$ is an invariant and so $\frac{\epsilon_{pq} A^p B^q}{\sqrt{(A^i A_i)(B^j B_j)}}$ is an invariant.

We define

$$\cos \theta = \frac{\epsilon_{pq} A^p B^q}{\sqrt{(A^i A_i)(B^j B_j)}}$$

as the cosine of the angle between vectors A^p and B^q . If $\epsilon_{pq} A^p B^q = A^p B_p = 0$, the vectors are called *orthogonal*.

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